HSD Secondary Mathematics Learning Progressions by Domain

Ratios and Proportional Relationships

Ratios and proportional relationships are foundational for further study in mathematics and science and useful in everyday life. Students use ratios in geometry and in algebra when they study similar figures and slopes of lines, and later when they study sine, cosine, tangent, and other trigonometric ratios in high school. Students use ratios when they work with situations involving constant rates of change, and later in calculus when they work with average and instantaneous rates of change of functions		
6th Grade	**7th Grade**	**8th Grade**
Understand and use ratios to solve problems.	Analyze proportional relationships and use them to solve problems.	Understand the connections between proportional relationships, lines and linear equations.
• Understand a ratio as a comparison of two quantities and represent these comparisons. • Understand the concept of a unit rate associated with a ratio, and describe the meaning of unit rate. • Create tables of equivalent ratios, find missing values in the tables and plot the pairs of values on the Cartesian coordinate plane. • Solve unit rate problems. • Solve percent problems. • Convert measurement units within and between two systems of measurement.	• Compute unit rates, including those that involve complex fractions, with like or different units. • Determine when two quantities are in a proportional relationship. • Identify and/or compute the constant of proportionality (unit rate). • Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation. • Recognize that the graph of any proportional relationship will pass through the origin. • Solve problems involving ratios, rates, percentages and proportional relationships.	• Graph proportional relationships. • Interpret the unit rate as the slope of the graph. • Compare two different proportional relationships. • Apply concepts of slope and y-intercept to graphs, equations and proportional relationships. • Explain why the slope (m) is the same between any two distinct points on a non-vertical line in the Cartesian coordinate plane. • Derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.

HSD Secondary Mathematics Learning Progressions by Domain

Number Sense and Operations/Number and Quantity

In Grades 6–8 students began to widen the possible types of number they can conceptualize on the number line. In Grade 8 they glimpse the existence of irrational numbers such as $\sqrt{2}$. In high school, they start a systematic study of functions that can take on irrational values, such as exponential, logarithmic, and power functions

6th Grade	7th Grade	8th Grade	Algebra 1	Algebra 2
Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	Apply and extend previous understandings of operations to add and subtract rational numbers.	Know that there are numbers that are not rational, and approximate them by rational numbers.	Extend and use properties of rational exponents.	Extend and use the relationship between rational exponents and radicals.
• Compute and interpret quotients of positive fractions. • Solve problems involving division of fractions by fractions. **Apply and extend previous understandings of numbers to the system of rational numbers.** • Use positive and negative numbers to represent quantities. • Locate a rational number as a point on the number line. • Understand that the absolute value of a rational number is its distance from 0 on the number line. • Extend prior knowledge to generate equivalent representations of rational numbers between fractions, decimals and percentages	• Apply and extend previous understandings of numbers to add and subtract rational numbers. • Add and subtract rational numbers. • Represent addition and subtraction on a horizontal or vertical number line. • Describe situations and show that a number and its opposite have a sum of 0 (additive inverses). • Understand subtraction of rational numbers as adding the additive inverse. • Determine the distance between two rational numbers on the number line is the absolute value of their difference. • Interpret sums and differences of rational numbers.	• Explore the real number system. • Know the differences between rational and irrational numbers. • Understand that all rational numbers have a decimal expansion that terminates or repeats. • Convert decimals which repeat into fractions and fractions into repeating decimals. • Generate equivalent representations of rational numbers.	• Explain how the meaning of rational exponents extends from the properties of integer exponents. • Rewrite expressions involving radicals and rational exponents using the properties of exponents. Limit to rational exponents with a numerator of 1.	• Extend the system of powers and roots to include rational exponents. • Create and recognize equivalent expressions involving radical and exponential forms of expressions. • Add, subtract, multiply and divide radical expressions. • Solve equations involving rational exponents and/or radicals and identify situations where extraneous solutions may result.

HSD Secondary Mathematics Learning Progressions by Domain

Number Sense and Operations/Number and Quantity Continued

6th Grade	7th Grade	8th Grade	Algebra 1	Algebra 2
Compute with non-negative multi-digit numbers, and find common factors and multiples.	Apply and extend previous understandings of operations to multiply and divide rational numbers.	Work with radicals and integer exponents.	Use units to solve problems.	Use complex numbers.
• Demonstrate fluency with division of multi-digit whole numbers. • Demonstrate fluency with addition, subtraction, multiplication and division of decimals. • Find common factors and multiples. • Find the greatest common factor (GCF) and the least common multiple (LCM). • Use the distributive property to express a sum of two whole numbers with a common factor as a multiple of a sum of two whole numbers.	• Apply and extend previous understandings of numbers to multiply and divide rational numbers. • Multiply and divide rational numbers. • Determine that a number and its reciprocal have a product of 1 (multiplicative inverse). • Understand that every quotient of integers (with non-zero divisor) is a rational number. • Convert a rational number to a decimal. • Understand that all rational numbers can be written as fractions or decimal numbers that terminate or repeat. • Interpret products and quotients of rational numbers by describing real-world contexts.	• Know and apply the properties of integer exponents to generate equivalent expressions. • Investigate concepts of square and cube roots. • Solve equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. • Evaluate square roots of perfect squares less than or equal to 625 and cube roots of perfect cubes less than or equal to 1000. • Recognize that square roots of non-perfect squares are irrational. • Express very large and very small quantities in scientific notation and approximate how many times larger one is than the other. • Use scientific notation to solve problems.	• Use units of measure as a way to understand and solve problems involving quantities. • Identify, label and use appropriate units of measure within a problem. • Convert units and rates. • Use units within problems. • Choose and interpret the scale and the origin in graphs and data displays. • Define and use appropriate quantities for representing a given context or problem. • Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	• Represent complex numbers. • Add, subtract, multiply and divide complex numbers. • Know and apply the Fundamental Theorem of Algebra.

HSD Secondary Mathematics Learning Progressions by Domain

Expressions, Equations and Inequalities

An expression is a phrase in a sentence about a mathematical or real-world situation. As with a facial expression, however, you can read a lot from an algebraic expression (an expression with variables in it) without knowing the story behind it, and it is a goal of this progression for students to see expressions as objects in their own right, and to read the general appearance and fine details of algebraic expressions.

6th Grade	7th Grade	8th Grade	Algebra 1	Algebra 2
Apply and extend previous understandings of arithmetic to algebraic expressions.	Use properties of operations to generate equivalent expressions.	Define, evaluate and compare functions.	Interpret and use structure.	Define and use logarithms.
• Describe the difference between an expression and an equation. • Create and evaluate expressions involving variables and whole number exponents. • Identify parts of an expression using mathematical terminology. • Evaluate expressions at specific values of the variables. • Evaluate non-negative rational number expressions. • Write and evaluate algebraic expressions. • Understand the meaning of the variable in the context of the situation. • Identify and generate equivalent algebraic expressions using mathematical properties.	• Apply properties of operations to simplify and to factor linear algebraic expressions with rational coefficients. • Understand how to use equivalent expressions to clarify quantities in a problem.	• Understand that a function assigns to each input exactly one output. • Determine if a relation is a function. • Graph a function. • Investigate the differences between linear and nonlinear functions. • Recognize that the graph of a linear function has a constant rate of change • Give examples of nonlinear functions. **Use functions to model relationships between quantities.** • Explain the parameters of a linear function based on the context of a problem. • Describe the functional relationship between two quantities from a graph or a verbal description.	• Interpret the contextual meaning of individual terms or factors from a given problem that utilizes formulas or expressions. • Analyze the structure of polynomials to create equivalent expressions or equations. • Choose and produce equivalent forms of a quadratic expression or equations to reveal and explain properties. • Find the zeros of a quadratic function by rewriting it in factored form. • Find the maximum or minimum value of a quadratic function by completing the square.	• Develop the definition of logarithms based on properties of exponents. • Use the inverse relationship between exponents and logarithms to solve exponential and logarithmic equations. • Use properties of logarithms to solve equations or find equivalent expressions. • Understand why logarithmic scales are used, and use them to solve problems.

HSD Secondary Mathematics Learning Progressions by Domain

Expressions, Equations and Inequalities Continued

6th Grade	7th Grade	8th Grade	Algebra 1	Algebra 2
Reason about and solve one-variable equations and inequalities.	Solve problems using numerical and algebraic expressions and equations.	Analyze and solve linear equations and inequalities and pairs of simultaneous linear equations.	Create equations that describe linear, quadratic and exponential relationships.	Solve equations and inequalities.
• Use substitution to determine whether a given number in a specified set makes a one-variable equation or inequality true. • Understand that if any solutions exist, the solution set for an equation or inequality consists of values that make the equation or inequality true. • Write and solve equations using variables to represent quantities, and understand the meaning of the variable in the context of the situation. • Solve one-step linear equations in one variable involving non-negative rational numbers. • Recognize that inequalities may have infinitely many solutions.	• Solve multi-step problems posed with rational numbers. • Convert between equivalent forms of the same number. • Assess the reasonableness of answers using mental computation and estimation strategies. • Write and/or solve linear equations and inequalities in one variable. • Write and/or solve equations of the form $x+p = q$ and $px = q$ in which p and q are rational numbers. • Write and/or solve two-step equations of the form $px + q = r$ and $p(x + q) = r$, where p, q and r are rational numbers, and interpret the meaning of the solution in the context of the problem.	• Solve linear equations and inequalities in one variable. • Create and identify linear equations with one solution, infinitely many solutions or no solutions. • Solve linear equations and inequalities with rational number coefficients, including equations and inequalities whose solutions require expanding expressions using the distributive property and combining like terms. • Analyze and solve systems of linear equations. • Graph systems of linear equations and recognize the intersection as the solution to the system.	• Create equations and inequalities in one variable and use them to model and/or solve problems. • Create and graph linear, quadratic and exponential equations in two variables. • Represent constraints by equations or inequalities and by systems of equations or inequalities, and interpret the data points as a solution or non-solution in a modeling context. • Solve literal equations and formulas for a specified variable that highlights a quantity of interest.	• Create and solve equations and inequalities, including those that involve absolute value. • Solve rational equations where numerators and denominators are polynomials and where extraneous solutions may result.

HSD Secondary Mathematics Learning Progressions by Domain

Expressions, Equations and Inequalities Continued

6th Grade	7th Grade	8th Grade	Algebra 1
Reason about and solve one-variable equations and inequalities.	Solve problems using numerical and algebraic expressions and equations.	Analyze and solve linear equations and inequalities and pairs of simultaneous linear equations.	Understand solving equations as a process, and solve equations and inequalities in one variable.
• Write an inequality of the form x > c, x < c, x ≥ c, or x ≤ c to represent a constraint or condition. • Graph the solution set of an inequality	• Write, solve and/or graph inequalities of the form px + q > r or px + q < r, where p, q and r are rational numbers.	• Explain why solution(s) to a system of two linear equations in two variables correspond to point(s) of intersection of the graphs. • Explain why systems of linear equations can have one solution, no solution or infinitely many solutions. • Solve systems of two linear equations.	• Explain how each step taken when solving an equation or inequality in one variable creates an equivalent equation or inequality that has the same solution(s) as the original. • Solve problems involving quadratic equations. • Use the method of completing the square to create an equivalent quadratic equation. • Derive the quadratic formula. • Analyze different methods of solving quadratic equations.
Represent and analyze quantitative relationships between dependent and independent variables.			
• Identify and describe relationships between two variables that change in relationship to one another. • Write an equation to express one quantity, the dependent variable, in terms of the other quantity, the independent variable. • Analyze the relationship between the dependent and independent variables using graphs, tables and equations and relate these representations to each other.			

HSD Secondary Mathematics Learning Progressions by Domain

Expressions, Equations and Inequalities Continued

Algebra 1	Algebra 2	Geometry
Solve systems of equations.	Solve general systems of equations and inequalities.	Translate between the geometric description and the equation for a conic section.
• Solve a system of linear equations algebraically and/or graphically. • Solve a system consisting of a linear equation and a quadratic equation algebraically and/or graphically. • Justify that the technique of linear combination produces an equivalent system of equations.	• Create and solve systems of equations that may include non-linear equations and inequalities.	• Derive the equation of a circle. • Derive the equation of a parabola given a focus and directrix.
Represent and solve linear and exponential equations and inequalities graphically.		Use coordinates to prove geometric theorems algebraically.
• Explain that the graph of an equation in two variables is the set of all its solutions plotted in the Cartesian coordinate plane. • Graph the solution to a linear inequality in two variables. • Solve problems involving a system of linear inequalities.		• Use coordinates to prove geometric theorems algebraically. • Prove the slope criteria for parallel and perpendicular lines and use them to solve problems. • Find the point on a directed line segment between two given points that partitions the segment in a given ratio. • Use coordinates to compute perimeters of polygons and areas of triangles and rectangles.

HSD Secondary Mathematics Learning Progressions by Domain

Expressions, Equations and Inequalities Continued

Algebra 1	Algebra 2
Perform operations on polynomials	Perform operations on polynomials and rational expressions.
- Add, subtract and multiply polynomials, and understand that polynomials follow the same general rules of arithmetic and are closed under these operations. - Divide polynomials by monomials.	- Extend the knowledge of factoring to include factors with complex coefficients. - Understand the Remainder Theorem and use it to solve problems. - Find the least common multiple of two or more polynomials. - Add, subtract, multiply and divide rational expressions. - Identify zeros of polynomials when suitable factorizations are available, and use the zeros to sketch the function defined by the polynomial.

HSD Secondary Mathematics Learning Progressions by Domain

Functions/Modeling

Functions describe situations in which one quantity is determined by another. The area of a circle, for example, is a function of its radius. When describing relationships between quantities, the defining characteristic of a function is that the input value determines the output value or, equivalently, that the output value depends upon the input value.

8th Grade	Algebra 1	Algebra 2
Define, evaluate and compare functions.	Understand the concept of a function and use function notation.	Use and interpret functions.
- Understand that a function assigns to each input exactly one output. - Determine if a relation is a function. - Graph a function. - Investigate the differences between linear and nonlinear functions. - Recognize that the graph of a linear function has a constant rate of change - Give examples of nonlinear functions.	- Understand that a function from one set (domain) to another set (range) assigns to each element of the domain exactly one element of the range. - Represent a function using function notation. - Understand that the graph of a function labeled f is the set of all ordered pairs (x, y) that satisfy the equation $y=f(x)$. - Use function notation to evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	- Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems. - Translate between equivalent forms of functions. **Create new functions from existing functions.** - Create new functions by applying the four arithmetic operations and composition of functions (modifying the domain and range as necessary). - Derive inverses of functions, and compose the inverse with the original function to show that the functions are inverses. - Describe the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for linear, quadratic, cubic, square and cube root, absolute value, exponential and logarithmic functions.

HSD Secondary Mathematics Learning Progressions by Domain

Functions/Modeling Continued

8th Grade	Algebra 1	Algebra 2
Use functions to model relationships between quantities.	Interpret linear, quadratic and exponential functions in terms of the context.	Use functions to model real-world problems
• Explain the parameters of a linear function based on the context of a problem. • Describe the functional relationship between two quantities from a graph or a verbal description.	• Using tables, graphs and verbal descriptions, interpret key characteristics of a function that models the relationship between two quantities. • Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes. • Determine the average rate of change of a function over a specified interval and interpret the meaning. • Interpret the parameters of a linear or exponential function in terms of the context.	• Create functions and use them to solve applications of quadratic and exponential function model problems.
	Analyze linear, quadratic and exponential functions using different representations.	
	• Graph functions expressed symbolically and identify and interpret key features of the graph • Translate between different but equivalent forms of a function to reveal and explain properties of the function and interpret these in terms of a context. • Compare the properties of two functions given different representations.	
	Build new functions from existing functions (limited to linear, quadratic and exponential).	
	• Analyze the effect of translations and scale changes on functions.	

HSD Secondary Mathematics Learning Progressions by Domain

Functions/Modeling Continued

8ᵗʰ Grade	Algebra 1	Algebra 2
Define, evaluate and compare functions.	**Understand the concept of a function and use function notation.**	**Use and interpret functions.**
• Understand that a function assigns to each input exactly one output. • Determine if a relation is a function. • Graph a function. • Investigate the differences between linear and nonlinear functions. • Recognize that the graph of a linear function has a constant rate of change • Give examples of nonlinear functions.	• Understand that a function from one set (domain) to another set (range) assigns to each element of the domain exactly one element of the range. • Represent a function using function notation. • Understand that the graph of a function labeled f is the set of all ordered pairs (x, y) that satisfy the equation $y=f(x)$. • Use function notation to evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	• Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems. • Translate between equivalent forms of functions.
Use functions to model relationships between quantities.	**Construct and compare linear, quadratic and exponential models and solve problems.**	**Geometry**
		Apply geometric concepts in modeling situations
• Explain the parameters of a linear function based on the context of a problem. • Describe the functional relationship between two quantities from a graph or a verbal description.	• Distinguish between situations that can be modeled with linear or exponential functions. • Describe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. • Construct linear, quadratic and exponential equations given graphs, verbal descriptions or tables.	• Use geometric shapes, their measures and their properties to describe objects. • Apply concepts of density based on area and volume in modeling situations. • Apply geometric methods to solve design mathematical modeling problems.
	Use arithmetic and geometric sequences.	
	• Write arithmetic and geometric sequences in recursive and explicit forms, and use them to model situations and translate between the two forms. • Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the set of integers. • Find the terms of sequences given an explicit or recursive formula.	

HSD Secondary Mathematics Learning Progressions by Domain

Geometry/Measurement

Geometry has two important streams that begin in elementary grades: understanding properties of geometric figures and the logical connections between them, and developing and using formulas to compute lengths, areas and volumes. A third stream, coordinate geometry, surfaces in Grade 5, gains importance in Grades 6–8, and mingles with algebra to become analytic geometry in high school

6th Grade	7th Grade	8th Grade	Geometry
Solve problems involving area, surface area and volume.	Draw and describe geometrical figures and describe the relationships between them.	Understand congruence and similarity using physical models, transparencies or geometry software.	Experiment with transformations in the plane.
• Find the area of polygons by composing or decomposing the shapes into rectangles or triangles. • Find the volume of right rectangular prisms. • Understand that the volume of a right rectangular prism can be found by filling the prism with multiple layers of the base. • Apply V = l * w * h and V = Bh to find the volume of right rectangular prisms. • Understand signs of numbers in ordered pairs as indicating locations in quadrants of the Cartesian coordinate plane.	• Solve problems involving scale drawings of real objects and geometric figures, including computing actual lengths and areas from a scale drawing and reproducing the drawing at a different scale. • Determine if provided constraints will create a unique triangle through construction. • Construct special quadrilaterals given specific parameters. • Describe two-dimensional cross sections of pyramids, prisms, cones and cylinders.	• Verify experimentally the congruence properties of rigid transformations. • Verify that angle measure, betweeness, collinearity and distance are preserved under rigid transformations. • Investigate if orientation is preserved under rigid transformations. • Understand that two-dimensional figures are congruent if a series of rigid transformations can be performed to map the pre-image to the image. • Describe a possible sequence of rigid transformations between two congruent figures. • Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates.	• Define angle, circle, perpendicular line, parallel line, line segment and ray based on the undefined notions of point, line, distance along a line and distance around a circular arc. • Represent transformations in the plane, and describe them as functions that take points in the plane as inputs and give other points as outputs. • Describe the rotational symmetry and lines of symmetry of two-dimensional figures. • Develop definitions of rotations, reflections and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments. • Demonstrate the ability to rotate, reflect or translate a figure, and determine a possible sequence of transformations between two congruent figures.
			Explain volume formulas and visualize relationships between 2D and 3D objects.
			• Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid and cone. • Use volume formulas for cylinders, pyramids, cones, spheres and composite figures to solve problems. • Identify the shapes of two-dimensional cross-sections of three-dimensional objects. • Identify three-dimensional objects generated by transformations of two-dimensional objects.

HSD Secondary Mathematics Learning Progressions by Domain

Geometry/Measurement Continued

6th Grade	7th Grade	8th Grade	Geometry
Solve problems involving area, surface area and volume.	Draw and describe geometrical figures and describe the relationships between them.	Understand congruence and similarity using physical models, transparencies or geometry software.	Experiment with transformations in the plane.
• Recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. • Recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. • Find distances between points with the same first coordinate or the same second coordinate. • Construct polygons in the Cartesian coordinate plane. • Represent three-dimensional figures using nets made up of rectangles and triangles. • Use nets to find the surface area of three-dimensional figures whose sides are made up of rectangles and triangles.	• Solve problems involving angle measure, area, surface area, and volume. • Analyze the relationships among the circumference, the radius, the diameter, the area and Pi in a circle. • Know and apply the formulas for circumference and area of circles to solve problems.	• Understand that two-dimensional figures are similar if a series of transformations (rotations, reflections, translations and dilations) can be performed to map the pre-image to the image. • Derive the sum of the interior angles of a triangle. • Explore the relationship between the interior and exterior angles of a triangle. • Construct and explore the angles created when parallel lines are cut by a transversal. • Use the properties of similar figures to solve problems.	• Develop the definition of congruence in terms of rigid motions. • Develop the criteria for triangle congruence from the definition of congruence in terms of rigid motions. • Prove theorems about lines and angles. • Prove theorems about triangles. • Prove theorems about polygons. • Construct geometric figures using various tools and methods.

HSD Secondary Mathematics Learning Progressions by Domain

Geometry/Measurement Continued

6th Grade	7th Grade	8th Grade	Geometry
Solve problems involving area, surface area and volume.	Apply and extend previous understanding of angle measure, area and volume.	Understand and apply the Pythagorean Theorem.	Understand similarity and prove theorems involving similarity.
• Recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. • Recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. • Find distances between points with the same first coordinate or the same second coordinate. • Construct polygons in the Cartesian coordinate plane. • Represent three-dimensional figures using nets made up of rectangles and triangles. • Use nets to find the surface area of three-dimensional figures whose sides are made up of rectangles and triangles.	• Use angle properties to write and solve equations for an unknown angle. • Understand the relationship between area, surface area and volume. • Find the area of triangles, quadrilaterals and other polygons composed of triangles and rectangles. • Find the volume and surface area of prisms, pyramids and cylinders. •	• Use models to demonstrate a proof of the Pythagorean Theorem and its converse. • Use the Pythagorean Theorem to determine unknown side lengths in right triangles in problems in two- and three-dimensional contexts. • Use the Pythagorean Theorem to find the distance between points in a Cartesian coordinate system. • Solve problems involving surface area and volume. • Understand the concept of surface area and find surface area of pyramids. • Understand the concepts of volume and find the volume of pyramids, cones and spheres.	• Construct and analyze scale changes of geometric figures. • Use the definition of similarity to decide if figures are similar and to solve problems involving similar figures. • Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. • Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. • Understand that side ratios in right triangles define the trigonometric ratios for acute angles. • Explain and use the relationship between the sine and cosine of complementary angles. • Use trigonometric ratios and the Pythagorean Theorem to solve right triangles. • Derive the formula $A = 1/2\ ab\ \sin(C)$ for the area of a triangle. **Understand formulas and theorems about circles.** • Prove that all circles are similar using similarity transformations. • Identify and describe relationships among inscribed angles, radii and chords of circles. • Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. • Derive the formula for the length of an arc of a circle. • Derive the formula for the area of a sector of a circle.

HSD Secondary Mathematics Learning Progressions by Domain

Data Analysis, Statistics and Probability

Until Grade 8, almost all of students' statistical topics and investigations have dealt with univariate data, e.g., collections of counts or measurements of one characteristic. Eighth graders apply their experience with the coordinate plane and linear functions in the study of association between two variables related to a question of interest. In High School, Students learn that formal inference procedures are designed for studies in which the sampling or assignment of treatments was random, and these procedures may not be informative when analyzing nonrandomized studies, often called observational studies.

6th Grade	7th Grade	8th Grade	Algebra 1	Algebra 2
Develop understanding of statistical variability.	Use random sampling to draw inferences about a population.	Investigate patterns of association in bivariate data.	Summarize, represent and interpret data.	Make inferences and justify conclusions.
• Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. • Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread and overall shape. • Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary from a single number.	• Understand that statistics can be used to gain information about a population by examining a sample of the population. • Understand that a sample is a subset of a population. • Understand that generalizations from a sample are valid only if the sample is representative of the population. • Understand that random sampling is used to produce representative samples and support valid inferences. • Use data from multiple samples to draw inferences about a population and investigate variability in estimates of the characteristic of interest.	• Construct and interpret scatter plots of bivariate measurement data to investigate patterns of association between two quantities. • Generate and use a trend line for bivariate data, and informally assess the fit of the line. • Interpret the parameters of a linear model of bivariate measurement data to solve problems. • Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. • Use relative frequencies calculated for rows or columns to describe possible association between the two variables.	• Analyze and interpret graphical displays of data. • Use statistics appropriate to the shape of the data distribution to compare center and spread of two or more different data sets. • Interpret differences in shape, center and spreads in the context of the data sets, accounting for possible effects of outliers. • Summarize data in two-way frequency tables. • Construct a scatter plot of bivariate quantitative data describing how the variables are related; determine and use a function that models the relationship	• Analyze how random sampling could be used to make inferences about population parameters. • Determine whether a specified model is consistent with a given data set. • Describe and explain the purposes, relationship to randomization and differences among sample surveys, experiments and observational studies. • Use data from a sample to estimate characteristics of the population and recognize the meaning of the margin of error in these estimates.

HSD Secondary Mathematics Learning Progressions by Domain

Data Analysis, Statistics and Probability Continued

6th Grade	7th Grade	Algebra 1	Algebra 2	Geometry
Summarize and describe distributions.	Draw informal comparative inferences about two populations.	Summarize, represent and interpret data.	Make inferences and justify conclusions.	Understand independence and conditional probability
• Display and interpret data. • Use dot plots, histograms and box plots to display and interpret numerical data. • Create and interpret circle graphs. • Summarize numerical data sets in relation to the context. • Report the number of observations. • Describe the nature of the attribute under investigation, including how it was measured and its units of measurement. • Give quantitative measures of center and variability, as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context of the data. • Analyze the choice of measures of center and variability based on the shape of the data distribution and/or the context of the data.	• Analyze different data distributions using statistical measures. • Compare the numerical measures of center, measures of frequency and measures of variability from two random samples to draw inferences about the population. • Investigate the probability of chance events. • Investigate the relationship between theoretical and experimental probabilities for simple events • Explain possible discrepancies between a developed probability model and observed frequencies. • Find probabilities of compound events using organized lists, tables, tree diagrams and simulations.	• Construct a linear function to model bivariate data represented on a scatter plot that minimizes residuals. • Construct an exponential function to model bivariate data represented on a scatter plot that minimizes residuals. • Interpret the slope (rate of change) and the y-intercept (constant term) of a linear model in the context of the data. • Determine and interpret the correlation coefficient for a linear association. • Distinguish between correlation and causation.	• Describe and explain how the relative sizes of a sample and the population affect the margin of error of predictions. • Analyze decisions and strategies using probability concepts • Evaluate reports based on data.	• Describe events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections or complements of other events. • Understand the definition of independent events and use it to solve problems. • Calculate conditional probabilities of events. • Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. • Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. • Recognize and explain the concepts of conditional probability and independence in a context. • Apply and interpret the Addition Rule for calculating probabilities. • Apply and interpret the general Multiplication Rule in a uniform probability model. • Use permutations and combinations to solve problems.